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Groupthink and the Blunder of the Gauges

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ABSTRACT

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TABLE OF CONTENTS

TABLE OF FIGURES

I. INTRODUCTION

Our primary objective is to illustrate the fallacies associated with the gauge concept in electromagnetism and suggest a more valid formulation. The primary fallacy is that the vector potential lacks physical meaning because of the freedom of choice of gauge. This number and nature of the fallacies indicates that the physics community is not immune from groupthink.

The orthodoxy of the gauge concept has been repeated verbatim from one electromagnetism textbook to another for generations. The Lorenz gauge is at the heart of this orthodoxy. As a reflection of the uncritical acceptance of this concept, all textbooks (until very recently) have attributed the concept to H.A. Lorentz rather than its rightful author, L. Lorenz [1]. The first two editions of Jackson's "Electrodynamics", for example, attribute this gauge to H.A. Lorentz. This error is corrected in the third edition[2]. It seems that the gauge concept itself has received a similar lack of attention.

II. DEMONSTRATIONS OF THE GAUGE FALLACIES

At present, the gauge concept is the centerpiece of electromagnetism. Electromagnetism is often considered a paradigm for gauge theories with the freedom to choose arbitrary values for a gauge presented as a convenient shortcut in problem solution. The evidence suggests that, on the contrary, the electromagnetic gauge concept is a source of numerous fallacies that also mask some important physics principles.

We begin with some elementary notes: According to the Helmholtz theorem, which we invoke throughout, any physically meaningful vector can be written as a sum of a gradient of a scalar and a curl of a vector. A gauge choice is required if one needs to obtain quantitative expressions for variables that are described only by a curl equation. One generally chooses a gauge that produces the least cumbersome form for relationships among variables, in analogy with the choice of zero for the Coulomb potential. The choice of zero for the gauge usually serves this purpose. Also, alternative choices of gauge cannot affect the underlying physics. These points are illustrated in the following.

A. Hidden Gauge

The magnetic field B is given by,

$$B = \nabla \times A. \tag{1}$$

According to the Helmholtz theorem, the general expression for the vector potential, A, is thus given by

$$A = \nabla \times F_A + \nabla \varphi_A. \tag{2}$$

Equation (1) defines A to within an arbitrary gradient function, $\nabla \varphi_A$, so a gauge choice is required. A non-zero divergence of a vector implies the existence of a scalar field associated with that vector. As we will see, one source of confusion in the literature is that vector potential is treated as the sole variable

1

requiring gauge choice. This confusion arises, in part, from the practice of lumping different variables under the same label. We now briefly review this standard practice, and provide an alternative approach where the variables are labeled to help eliminate this confusion.

Faraday's law is originally expressed as a line integral of the induced field, E_I , around a closed path, which leads to the relationship, $\nabla \times E_I = -\partial B / \partial t$. Since the Coulomb field is derivable from a scalar potential, $E_C = -\nabla \varphi_C$, when these two electric fields are present together the fields can be summed into a total, $E = E_I + E_C$, giving $\nabla \times E = -\partial B / \partial t$. Applying Eq.(1) gives $\nabla \times (E + \partial A / \partial t) = 0$. Thus, $E + \partial A / \partial t = \nabla \varphi$, with $\nabla \varphi$ representing the gradient of a general scalar, so that E originates from a scalar and a vector potential,

$$E = -\nabla \varphi - \partial A / \partial t . \tag{3}$$

This leaves the impression that there is only one variable requiring a gauge choice in order to give a quantitative definition for E . If one preserves the distinction among variables, it is clear that there are two gauges to consider. Adhering to the original form of the Faraday law, which relates specifically to the induced field, E_I , one obtains

$$\nabla \times E_I = -\partial B / \partial t = -\nabla \times \partial A / \partial t , \tag{4}$$

so that,

$$\nabla \times (E_I + \partial A / \partial t) = 0 . \tag{5}$$

Again, the curl defines the quantity in parenthesis to within an arbitrary gradient of a scalar function, φ_I , associated with the definition of E_I. So the general expression for E_I is

$$E_I = -\partial A / \partial t - \nabla \varphi_I , \tag{6}$$

and the total field,

$$E = E_C + E_I = -\nabla (\varphi_C + \varphi_I) - \partial A / \partial t , \tag{7}$$

with A given in general form by Eq.(2). Comparing Eqs.(3) and (7) shows that φ is not the Coulomb potential as conventionally assumed, but is generally the sum of two scalar fields, $\varphi = \varphi_C + \varphi_I$. So, substituting the Coulomb potential, φ_C , in place of φ implies the hidden gauge choice $\varphi_I = 0$. Similarly, it can be seen from Eq.(6) that the standard practice of employing

$$E_I = -\partial A / \partial t \tag{8}$$

implies the same hidden gauge choice, $\varphi_I = 0$. A general observation, which can be verified by reviewing the literature, is that the hidden gauge choice is made universally in electromagnetism in one of the two ways described above, so $\varphi_I = 0$ is the implicit "standard gauge" in electromagnetism.

We can illustrate how the hidden gauge leads to the conventional formalism using Gauss' law for dynamic fields, which is given by

$$\nabla \cdot E = \nabla \cdot (E_C + E_I) = \rho/\varepsilon. \tag{9}$$

Inserting Eq.(6) into Eq.(9) gives the general expression of Gauss' law in terms of the two gauge choices,

$$\nabla \cdot E = \nabla \cdot (-\nabla(\varphi_C + \varphi_I) - \partial A/\partial t) = \rho/\varepsilon. \tag{10}$$

It follows from Eq.(10) that the free choice of one gauge precludes the free choice of the second. We will discuss this further in a later section.

Applying the hidden gauge choice, $\varphi_I = 0$, gives us the conventional expression of the dynamic Gauss' law,

$$\nabla \cdot (-\nabla\varphi_C - \partial A/\partial t) = \rho/\varepsilon. \tag{11}$$

Another illustration of the implicit use of the hidden gauge is the vector potential wave equation. The general expression for wave equation for the vector potential in terms of the two arbitrary gauge choices is obtained using Ampere's law,

$$\nabla \times B = \nabla \times \nabla \times A = \mu J_{TOT} = \mu(J_T + \varepsilon \, \partial E / \partial t), \tag{12}$$

where J_T is the true current. Applying Eq.(6) to Eq.(12) gives the general wave equation in terms of the two gauges,

$$\nabla(\nabla \cdot A) - \nabla^2(A) = \mu J_T - \mu\varepsilon\partial\nabla(\varphi_C + \varphi_I)/\partial t - \mu\varepsilon\partial^2 A/\partial t^2. \tag{13}$$

Again, we see that applying the standard gauge, $\varphi_I = 0$, gives the familiar standard form for the wave equation for A,

$$\nabla^2(A) - \mu\varepsilon\partial^2 A/\partial t^2 = -\mu J_T + \nabla(\nabla \cdot A + (\partial\varphi_C/\partial t)/c^2). \tag{14}$$

B. Hidden Law of Physics

The usual derivation of the dynamic form of Gauss' law (Eq.(9)) involves nothing more than plugging the sum of the dynamic fields into the static expression for the Coulomb field. That is not justified, and there is much more to it. Equation (9) contains a hidden law. Because of retardation effects, the Coulomb field no longer propagates instantaneously, so it no longer obeys Gauss' law. Equation (9) really says that the dynamic Coulomb field induces a self-correcting, non-solenoidal component to the E_I field so that the total electric field obeys Gauss' law. Consequently, the total E, in effect, appears to propagate

instantaneously from a central source or a distribution of central sources. The basic example of this phenomenon is the pair of Lienard-Weichert potentials for a moving point charge; for a moving charge, neither E_C nor E_I is radial, but the sum is radial, so that the E field obeys Gauss' law and appears to propagate instantaneously. This effect is arguably the most fascinating phenomenon in electromagnetism and is the key to preserving the continuity equation. Unfortunately, it is buried beneath layers of gauge fallacies.

We stated above that there is more to the derivation of the dynamic form of Gauss' law than simply substituting the total dynamic electric field into the static form of Gauss' law. It really derives from the need to preserve the continuity equation. This can be shown in the standard gauge, for example, by combining the time derivative of Gauss' law Eq.(11) with the divergence of Ampere's law given in Eq.(12). The result is

$$\nabla \cdot J_T + \partial\rho/\partial t = 0. \tag{15}$$

So the real basis for the dynamic form of Gauss' law is the requirement of charge conservation.

The hidden law in the dynamic form of Gauss' law is expressed as: *A dynamic Coulomb field induces a self-correcting scalar component for the induced field so that the total electric field obeys Gauss' law.* Implicit in this law is that there are no induced scalar fields in the absence of dynamic Coulomb fields ($\varphi_I = 0$). Faraday's law defines E_I via a closed line integral so any scalar component is left undefined. This hidden law actually supplements Faraday's law to allow a complete definition of E_I (completely defined by curl and divergence). This hidden law is unrelated to any gauge choice. It can be given precise mathematical expression by invoking the scalar wave equation for the Coulomb field as a law of physics,

$$\nabla^2\varphi_C - (\partial^2\varphi_C/\partial t^2)/c^2 = -\rho/\varepsilon, \tag{16}$$

and comparing this with the general expression for Gauss' law, Eq.(9). By inspection it is seen that

$$\nabla \cdot E_I = -(\partial^2\varphi_C/\partial t^2)/c^2 \tag{17}$$

is a general requirement. Equation (17) applies under all circumstances. It is independent of any gauge choice. In the absence of dynamic fields, $\nabla \cdot E_I = 0$, which is consistent with the hidden law that requires there be no induced scalar fields in the absence of dynamic Coulomb fields. All of this indicates that Eq.(17) be considered a missing Maxwell equation.

We next consider this hidden law in the standard gauge ($\varphi_I = 0$). It follows directly from Eqs.(8) and (17) that

$$\nabla \cdot A = \nabla^2\varphi_A = -(\partial\varphi_C/\partial t)/c^2. \tag{18}$$

Equation (18) has exactly the same form as the Lorenz condition, but here it is more than a condition. It derives from a law of physics. It reflects a causal relationship describing precisely how the dynamic φ_C induces φ_A. Combining Eq.(18) with Eq.(1) defines the vector potential A completely in all circumstances in the standard gauge. Note that if one initially uses Eq.(18) as the gauge choice, then it

4

follows from Eq.(10) and (16) that $\varphi_I = 0$, so that choice is equivalent to the standard gauge. These serve as an example of a consistent set of gauge choices.

For completeness, we note that the counterpart to Eq.(18) in the Coulomb gauge ($\nabla \cdot A = 0$) is obtained in the same manner, giving

$$\nabla^2 \varphi_I = -(\partial^2 \varphi_C / \partial t^2) / c^2. \tag{19}$$

Equation (18) describes how the dynamic Coulomb field induces the scalar φ_I. At this point, we see that by explicitly adopting the hidden, standard gauge and adding Eq.(8) and (18) to Maxwell's equations, one has a complete set of familiar fundamental equations. There is no need for further discussion of the electromagnetic gauge concept. One can initially fix the gauge ($\varphi_I = 0$) in a footnote, and forget it.

We continue the discussion of the details of the gauge concept, however, in order to address the other major fallacies. Thus far we have considered the first two of the major fallacies: there are two electromagnetic gauges, not one, and Eq.(18) is not merely a condition but is derived from a hidden universal law of physics, and represents a fundamental physical requirement for the vector potential, A, defined by $\varphi_I = 0$.

C. In the Absence of Dynamic Coulomb Fields

We now turn to the cases where dynamic Coulomb fields are absent (i.e., where there are no retarded Coulomb fields). This will also serve as background for discussing Konopinski's paper [3,4] on the meaning of the vector potential. Assume the standard gauge ($\varphi_I = 0$). In regions where Coulomb fields are absent, the law of physics expressed by Eq.(18) gives $\nabla \cdot A = 0$. This coincides with the choice of Coulomb gauge (or radiation gauge) which is conventionally invoked in such cases. On the other hand, if one begins with the Coulomb gauge, $\nabla \cdot A = 0$, then Gauss' law (Eq.(10)) requires $\varphi_I = 0$. Thus, both gauge choices are compatible and both give $\nabla \cdot E_I = 0$, and $E_I = -\partial A / \partial t$.

Examining the real reason for this compatibility shows another major fallacy. There is a conflict between the gauge concept and physical reality here which can be seen from the hidden law of physics discussed above: in the absence of dynamic Coulomb fields, there can be no induced scalar component for E_I so $\varphi_I = 0$. Thus, $\nabla \cdot E_I = -\partial(\nabla \cdot A)/\partial t = 0$ is a fundamental physical requirement. Consequently, the gauge concept itself is not applicable in the absence of dynamic Coulomb fields for the simple reason that one does not have free choice of $\nabla \cdot A$. Note that since $\partial(\nabla \cdot A)/\partial t = 0$ holds for all times, (e.g., during the energizing of a solenoid) the only physically meaningful option is $\nabla \cdot A = 0$. Thus, the "Coulomb gauge" is always invoked erroneously since it is always applied in the absence of dynamic Coulomb fields where there is no free choice. (We give a more general proof that $\nabla \cdot A = 0$ is a physical requirement in the absence of Coulomb fields in section F, below)

We now consider the physical meaning of the vector potential as developed by Konopinski in his challenge to the orthodox view that A has no physical meaning. Using the infinite solenoid, (as used in the Bohm-Aharonov effect to show the reality of A in quantum mechanics) he demonstrates that the

vector potential was always unique and had physical meaning as the field momentum of a unit charge in a magnetic field for the solenoid. (Note that the field momentum is dependent upon gauge choice in the general case.) Konopinski also gives an operational definition for A, in analogy with the operational definition of the potential of a static Coulomb field. This gives A measurability. Thus, the earlier experimental demonstration of the Bohm-Ahronov effect was unnecessary.

A comment on Konopinski's [3,4] basic assumptions: He specifically makes the usual assumption that the potential in Eq.(3) is a Coulomb potential. This implies the hidden gauge assumption, φ_I =0. He also explicitly assumes that the Coulomb gauge applies. This assumption overlooks the larger issue that in the absence of any Coulomb fields, the gauge concept itself is not applicable. So the real reason that Konopinski can give unique physical meaning to this vector potential is the same reason that the Bohm-Aharonov effect gives a unique physical meaning to A: A is uniquely defined because the gauge concept does not apply in the absence of dynamic Coulomb fields.

D. In the Presence of Dynamic Coulomb Fields

A gauge choice is only required in the presence of retardation effects associated with dynamic Coulomb fields. As we have discussed, for such cases there are two available gauges to consider. Return to the standard gauge wave equation (φ_I =0),

$$\nabla^2(A) - \mu\varepsilon\partial^2 A/\partial t^2 = -\mu J_T + \nabla(\nabla \cdot A + (\partial\varphi_C/\partial t)/c^2). \tag{14}$$

Applying Eq.(18) has the effect of combining the Coulomb source term and the scalar component of A giving the familiar form for the wave equation in the standard gauge,

$$\nabla^2 A - \mu\varepsilon\,\partial^2 A/\partial t^2 = -\mu J_T . \tag{20}$$

Note that the induced scalar potential φ_A that exists in this gauge must also obey the wave equation. Substituting Eq.(2) into Eq.(19) shows,

$$\nabla^2\varphi_A - (\partial^2\varphi_A/\partial t^2)/c^2 = 0. \tag{21}$$

Gauss' law now takes on the form (φ_I =0),

$$\nabla^2(\varphi_C + \partial\varphi_A/\partial t) = \nabla^2\varphi = -\rho/\varepsilon . \tag{22}$$

Eq. (22) says that these two dynamic scalar fields add in such a manner as to produce a net quasi-static scalar field.

It is instructive to consider the corresponding Coulomb gauge ($\nabla \cdot A = 0$) expression for Eq.(22). This expression is obtained directly from Eq.(10),

$$\nabla^2(\varphi_C + \varphi_I) = \nabla^2\varphi = -\rho/\varepsilon \tag{23}$$

Note the similarity of Gauss' law in the two gauges, Eqs.(21) and (22). Both express the same physics contained in the hidden law but in different forms. Both say that the sum of the two dynamic scalar fields is a quasi-static field. In other words, φ is an instantaneously propagating field. In the standard gauge, the real induced scalar field is assigned completely to the vector potential, via φ_A, while in the Coulomb gauge, the real induced scalar field is assigned directly to E_I via φ_I.

We can now complete the set of equations with the Coulomb gauge expression for vector potential wave equation. Substituting $\nabla \cdot A = 0$ into the general form, Eq.(13), gives,

$$\nabla^2 A - \partial^2 A / \partial t^2 = -\mu J_T + \mu\varepsilon \ \partial\nabla(\varphi_C + \varphi_I)/\partial t . \tag{24}$$

As with the other relationships, the two vector wave equations, Eqs.(20) and (24), in their respective gauges describe the same physics, but in different forms. The difference is accounted for by the fact that the vector potential in Eq.(24) is solenoidal, while that in Eq.(20) is not. Equation (24) shows nicely that the vector potential is solenoidal if one adds the net displacement current to the true current in order to form a source that is a closed current loop. Note that the displacement current arises from the time derivative of $\nabla\varphi$ in Eq.(24), which obeys Gauss' law at all times. Thus, it is an instantaneously propagating longitudinal field, producing an instantaneously propagating displacement current.

From the above discussion it is clear that current continuity is at the root of the hidden law. The propagation of displacement currents from scalar fields are longitudinal, extending the longitudinal path of the true current, J_T, to form a closed loop.

The reader can confirm that both wave equations, Eqs. (20) and (24), express the same physics: the divergence of both returns the continuity equation, and the curl of both returns the wave equation for the magnetic field, B. (The reason that different vector potentials give the same B is that scalar components contribute nothing to B.)

E. The Peculiarity of the Coulomb Gauge

Jackson's textbook (3rd edition) [2,5] offers a demonstration that the Coulomb gauge exhibits the "peculiar" requirement that dynamic Coulomb fields propagate instantaneously. The problem is that this demonstration clashes with relativity and, specifically, with the fundamental law of physics embodied by the wave equation for Coulomb fields, Eq.(16).

We reproduce Jackson's demonstration here. Apply the Coulomb gauge, $\nabla \cdot A = 0$, to the standard gauge expression, Eq.(11), to give,

$$\nabla^2\varphi_C = -\rho/\varepsilon . \tag{25}$$

This equation indeed requires that Coulomb fields propagate instantaneously; it is an absurdity, however, arising out of the simultaneous selection of the hidden gauge $\varphi_I = 0$ and the conflicting Coulomb gauge $\nabla \cdot A = 0$. One cannot select conflicting gauge choices without violating the laws of physics.

F. A Picture of Reality

Let us now reconsider the wave equations in the two gauge choices from a clearer perspective. The vector potential that appears in Coulomb gauge (Eq.(24)) form of the wave equation is actually the total vector potential, A_{TOT}. It originates from the closed current loop, J_{TOT}, comprised of true currents propagating instantaneously within conductors and displacement currents arising from the instantaneously propagating longitudinal fields $\nabla\varphi$.

The standard gauge wave equation, Eq.(20), describes a different vector potential. It originates from the true current source J_T, rather than the total J_{TOT}. Hence, the vector potential A in that equation is a component of A_{TOT}. Consequently A is not solenoidal, and it contains a scalar source term. The point is that if A_{TOT} is the total vector potential that arises from a closed current loop, it must be solenoidal ($\nabla \cdot A_{TOT} = 0$). This can be proven using Eq.(12), rewritten as,

$$\nabla(\nabla \cdot A_{TOT}) - \nabla^2 A_{TOT} = \mu J_{TOT} . \tag{26}$$

It follows from taking the divergence of both sides of Eq.(26) that the continuity equation for a closed loop $\nabla \cdot J_{TOT} = 0$, requires $\nabla \cdot A_{TOT} = 0$. In the absence of Coulomb fields, A must arise from a closed loop of either J_T or displacement current $J_D = \varepsilon \, \partial E / \partial t$. Thus, $A = A_{TOT}$ and $\nabla \cdot A = \nabla \cdot A_{TOT} = 0$ is a requirement of the continuity equation. There is no choice in the matter, so, as we illustrated in a different way earlier, the conventionally invoked "Coulomb gauge" and "radiation gauge" are always invoked erroneously.

The same is true for our Coulomb gauge labeling of Eq.(24). The choice of $\nabla \cdot A = 0$ produces basic equations for the real, physically meaningful vector potential, A_{TOT}. If our objective is to obtain basic equations for the real, physically meaningful vector potential, A_{TOT}, there is no choice.

By the same token, the familiar vector potential, A, associated with the standard gauge choice, $\varphi_I = 0$, has precise physical meaning as the component of A_{TOT} arising from J_T. The familiar vector potential, A, is completely defined because its divergence is given by Eq.(18). Referring to Eq.(2), A has a scalar component. It follows from these examples that the freedom of choice of gauge does not imply undefined vector potentials, it implies freedom to choose different vector potentials, which is an entirely different matter

All of these points are summarized in the schematic of Figure 1 which compares the two general categories of problems one encounters in electromagnetism. The case where dynamic Coulomb fields are absent is shown on the left, and the case where Coulomb fields are present is given on the right. The schematic on the left corresponds to the case where the real current, J_T, forms a closed loop so that the gauge concept does not apply ($A_{TOT} = A$). For completeness include the equivalent case where the closed loop is comprised entirely of a displacement current corresponding to the physics of electromagnetic radiation ("radiation gauge").

The schematic on the right reflects the special case where both types of current exist together to form a closed loop ($A_{TOT} \neq A$). The overall circuit geometry is identical in both cases so A_{TOT} is the same in both. Perhaps this picture is worth more than all the words we offer here. It should be clear from this diagram that $\nabla \cdot A_{TOT} = 0$ in both cases, so the vector potential A_{TOT} is unique and completely defined in both. The side by side comparison also makes the point that inserting a capacitor into a circuit cannot transform the vector potential from a real, measurable quantity on the left to a mere mathematical convenience on the right.

G. <u>The Lorenz Transformation</u>

We stated in the introduction that the Lorenz transformation is the centerpiece of electromagnetism. We examine this in light of the above analyses. In terms of the Lorenz transformation function, ψ , this transformation is given as,

$$A^{'} = A + \nabla \psi; \quad \varphi^{'} = \varphi - \partial \psi / \partial t . \tag{27}$$

The freedom to define different vector potentials in terms of this transformation is justified on the basis that it leaves the total E unchanged. This same transformation can be reconsidered in terms of the two gauge choices so that

$$A^{'} = A + \nabla \varphi_A; \quad \varphi^{'}_{I} = \varphi_I + \partial \varphi_A / \partial t . \tag{28}$$

We can see that ψ has real physical meaning originating from the real scalar potential induced by the dynamic φ_C . Equation (28) describes exactly how the potentials, φ_I and φ_A can be freely distributed while preserving the net induced scalar field and the net electric field, E .

It is clear from the discussion presented to this point that the Lorenz transformation offers a misleading set of choices of vector potentials. Our focus has been on the two simplest gauge choices, $\varphi_A = 0$, and $\varphi_I = 0$. These provide mathematical expressions that relate to physically meaningful variables, A , A_{TOT} , and E_I which suffice for problem solving.

This existence of this range of choices in the special case where dynamic Coulomb fields are present cannot justify the general conclusion that vector potentials lack meaning. The vector potential A_{TOT} , for example, is real and exists in nature regardless of the fact that one can choose a gauge that doesn't relate to that vector potential. Different gauge choices define different vector potentials; they do not imply different definitions of the same vector potential, as often claimed. Freedom of choice does not imply that the vector potential is meaningless. To reinforce this point, the real reason that Equations (8) and (18) have proven so useful is that they apply to a real, physically meaningful, vector potential, the standard A .

The belief that electromagnetism is the paradigm for contemporary gauge theories is based entirely on freedom of choice offered by the Lorenz transformation. It is fair to question this characterization given that the need for a choice of gauge only exists in the special case where dynamic fields are present. Even in that special case, if one wants a set of basic equations that apply to a specific physically meaningful

vector potential, there is no choice. It is also fair to ask if this is the paradigm, how many other fundamental laws of physics are being masked by the contemporary gauge theories.

III. CONCLUDING REMARKS

The gauge-related fallacies evidently originate from a combination of carelessness, failure to distinguish among variables. In the standard formalism, the labels E, A, and φ are each commonly used to represent different variables at different times. We summarize the major items relating to gauge fallacies in the following:

a. The dynamic form of Gauss' law contains the hidden law of physics, $\nabla \cdot E_I = (\partial^2 \varphi_C / \partial t^2)/c^2$. This is one of the universal laws of electromagnetism and is needed to supplement Faraday's law of induction in order to permit a full definition of E_I. It can be viewed as a missing Maxwell equation.

b. In the absence of dynamic Coulomb fields, the gauge concept is invalid. A is uniquely defined and $\nabla \cdot A = 0$ is a physical requirement, so the Coulomb gauge is always applied incorrectly.

c. In the presence of dynamic Coulomb fields, the gauge concept is valid but there are two gauge options, not one. The standard formalism for electromagnetism is expressed in terms of a hidden gauge ($\varphi_I = 0$). Absurdities occur when conflicting gauge choices are exercised.

d. The improper interpretation of the Lorenz transformation is the primary source of the fallacy that the vector potential is only a mathematical convenience. It describes a range of choices of real physically meaningful vector potentials.

e. The Lorenz condition is much more than a condition. It is a precise statement of the hidden law of physics applicable to the vector potential, A, defined by the gauge choice, $\varphi_I = 0$.

f. The vector potential is not just a mathematical convenience. As shown by Konopinski, it has both unique physical meaning and measurability in the absence of dynamic Coulomb fields.

g. The Coulomb gauge does not require that Coulomb fields propagate instantaneously. That erroneous belief originates from the simultaneous adoption of conflicting gauge choices.

The simplicity of the demonstrations of these fallacies indicates that groupthink has subverted the norms of the scientific method regarding the gauge concept. Our experience with groupthink in this matter shows that it arises from the assumption that the electromagnetic gauge concept has been properly vetted by generations of distinguished scientists. This may be the greatest of the fallacies. The pervasiveness of these fallacies and their longevity indicate that the present gauge orthodoxy is the physics blunder of the ages.

REFERENCES

1. L. Lorenz, On the identity of vibrations of light with electrical currents, Philos. Mag. **34**, 287, 1867.

2. J.D. Jackson, Classical Electrodynamics, 3rd Ed., John Wiley and Sons, New York, 1999.

3. E.J. Konopinski, What the electromagnetic vector potential measures, Am. J. Phys., **46** (5) 499-502, 1978.

4. P.J.Cote and M.A. Johnson, Comments on what the electromagnetic vector potential measures by E.J.Konopinski, arXiv:1012.3718.

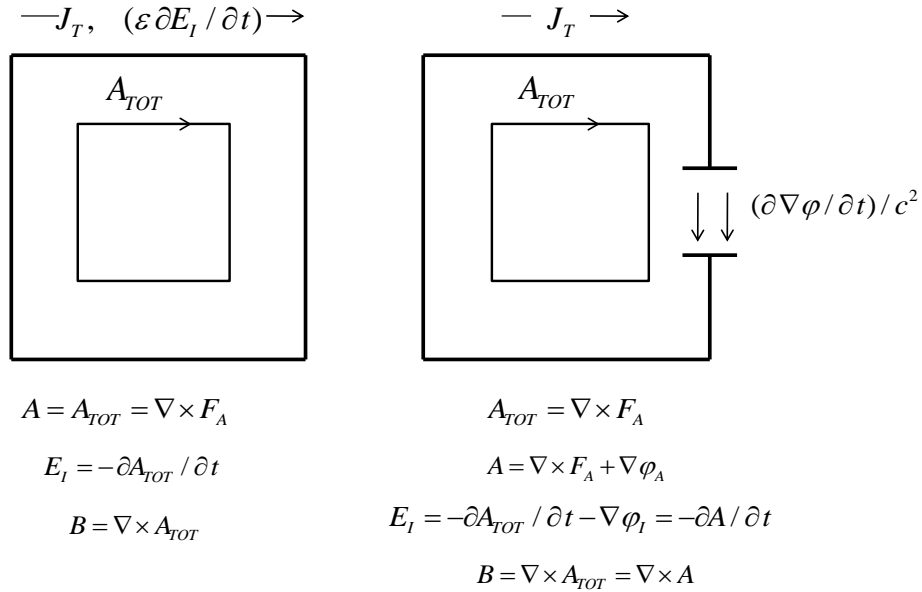5. P.J. Cote and M.A. Johnson, On the peculiarity of the Coulomb gauge, arXiv:0906.4752.

Figure 1. Schematics of the two classes of problems encountered in electromagnetism. The schematic on the left corresponds to closed current loops comprised of either true currents or displacement currents; there are no dynamic Coulomb fields so there are no induced scalar fields and the gauge concept does not apply. The schematic on the right corresponds to the case where dynamic Coulomb fields are present so induced scalar fields exist and gauge choices are required. The two choices for A and A_{TOT} are illustrated.